$\mathbb{F}_2$

$$P = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & & \\ 0 & & \\ 0 & & \end{matrix}$$

$P = (-A \ \ I_n)$

$G = \begin{pmatrix} I_n \\ A \end{pmatrix}$

$A^{100} \to SDS^{-1} = A$

$A^{100} = (SDS^{-1})^{100} = SD^{100}S^{-1}$

$\begin{pmatrix} \cdot^{100} & & \\ & \cdot^{100} & \\ & & \cdot^{100} \end{pmatrix}$

$\varphi(f)$

$f \in \mathbb{R}[x]$

$f = x^2 + x + 1$

$f(x^2) = (x^2)^2 + x^2 + 1$

$x^4 + x^2 + 1$

$\varphi : f \to f(2) \cdot x^2 + f'(x)$

$f = a_0 + a_1 x + x^2$

$\ker(\varphi)$

$\langle v, w \rangle \cdots$

$B = \{1, x, x^2\}$

$A = \Big(\ 1 \quad 2 \quad 4$

11b)

$B = \{x^2-1, x^3-4x, x^4-1\}$

$\Rightarrow$ Alle Vektoren in B sind lin. unabh.

$\lambda(x^2-1) + \mu\cdot(x^3-4x) + \alpha(x^4-1) = 0 \iff \lambda, \mu, \alpha = 0$

$\lambda^2 - \lambda + \mu x^3 - 4\mu x + \alpha x^4 - \alpha = 0$

$-4\mu x + \lambda^2 + \mu x^3 + \alpha x^4 - \underbrace{(\lambda+\alpha)}_{0} = 0$

$\lambda = -\alpha$

$-4\mu x + \alpha^2 + \mu x^3 + \alpha x^4$

Wenn α und $\mu \neq 0$, wäre das Polynom nicht 0

$\Rightarrow \alpha, \mu = 0 \Rightarrow \lambda = 0$

$f = \sum_{i=0}^{4} a_i x^i \in U$

$0 = f(1) + f(-1) = \sum_{i=0}^{4} a_i + \sum_{i=0}^{4} a_i \cdot (-1)^i = 2a_0 + 2a_2 + 2a_4 = 0$

$0 = f(2) - f(-2) = \sum_{i=0}^{4} a_i \cdot 2^i - \sum_{i=0}^{4} a_i \cdot (-2)^i = 0 = 4a_1 + 16a_3 = 0$

$\underline{2a_0 + 2a_2 + 2a_4} = \underline{4a_1 + 16a_3}$

$$a_0 = -a_2 - a_4 \qquad a_1 = -4a_3$$

$$f = \underline{(-a_2} \, \underline{-a_4)} - \underline{4a_3 x} + \underline{a_2 x} + \underline{a_3 x^2} + a_4 \underline{x^4} =$$

$$a_4 \cdot (x^4 - 1) + a_3 \cdot (-4x + x^2) + a_2 \cdot (x - 1)$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\lambda_1 \qquad \operatorname{rng}(A - \lambda_1 \cdot I_n)$$

$$a_0 = 2 \qquad a_1 = -1 \qquad a_{n+1} = -a_n + 6a_{n-1}$$

$$a_{n+1} = -a_n + 6a_{n-1}$$
$$a_n = a_n$$

$$\begin{pmatrix} a_{n+1} \\ a_n \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} a_{n+1} \\ a_n \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix} = \underline{\begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^n \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}$$

$$(0 \quad 1) \cdot \begin{pmatrix} a_{n+1} \\ a_n \end{pmatrix} = a_n$$

$$a_n = (0 \quad 1) \cdot \underbrace{\begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}^n}_{SDS^{-1}} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} =$$

$$a_n = (0 \quad 1) \cdot S D^n S^{-1} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\langle v, w \rangle$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 2 & 0 & 2 \\ 1 & 2 & 3 & 3 \end{array}\right) \begin{array}{l} \\ -2 \\ -1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right) -1$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right) \Rightarrow x_3 = \frac{1}{3} \quad x_2 = \frac{1}{3}$$

$$x_1 = -\frac{1}{3}$$

$$\varphi: \underline{ax^3 + bx^2 + cx + d} \mapsto \underline{\begin{pmatrix} a+b \\ c \\ c-a \end{pmatrix}}$$

$$E_1 = \{1, x, \underline{x^2}, \underline{x^3}\} \qquad E_2 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\underline{M^{E_1}_{E_2}(\varphi) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & \underline{0} & -1 \end{pmatrix}} \qquad C = \left\{ \underline{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}, \underline{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} \right\}$$

$$\begin{matrix} 1 \\ 0 \\ -1 \end{matrix} \qquad 1 \qquad 0 \qquad -1$$

$$M^{E_1}_{C} = \begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{pmatrix} \qquad \begin{matrix} -1 \\ 1 \\ 1 \end{matrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$\langle v_1, v_2 \rangle = \langle v_3, v_4 \rangle$

$v_3 \in \langle v_1, v_2 \rangle$

$v_4 \in \langle v_1, v_2 \rangle$

<u>v_3, v_4 lin abh.</u>

→ wenn ja: sind v_1, v_2 lin abh.

→ wenn ja $\langle v_1, v_2 \rangle = \langle v_3, v_4 \rangle$

$$\underline{M^{B'}_{B'}(\varphi)} = T^{B}_{B'} \cdot \underline{M^{B}_{B}(\varphi) \cdot T^{B'}_{B}}$$

$$M^{B}_{B}(\varphi)$$

$$T^{B}_{B'} \qquad T^{B}_{B'}$$

$A = SDS^{-1}$

$A^{n} = SD^{n}S^{-1}$

$\varphi : \mathbb{R}[x] \mapsto \mathbb{R}[x]$

$\downarrow$

$M_B^B(\varphi) =$