$$A = \begin{pmatrix} 5 & 12 \\ 12 & -2 \end{pmatrix}$$

$$\chi_A(\lambda) = (\lambda+11)(\lambda-14)$$

$$E_{14}: \quad \mathrm{Kern}\left(\begin{pmatrix} -9 & 12 \\ 12 & -16 \end{pmatrix}\right) = \mathrm{Kern}\left(\begin{pmatrix} -3 & 4 \\ 0 & 0 \end{pmatrix}\right) = \left\langle \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right\rangle$$

$$E_{-11}: \left\langle \begin{pmatrix} 4 \\ 3 \end{pmatrix}, v \right\rangle = 0 \qquad 4 \cdot v_1 + 3 \cdot v_2 = 0$$

$$v_1 = 3 \quad v_2 = -4 \quad (\text{bspw.})$$

$$\Rightarrow \left\langle \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right\rangle$$

$$\Rightarrow \mathbb{R}^2 \text{ wird aufgespannt von } \left\{ \frac{1}{\sqrt{4^2+3^2}} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \frac{1}{5} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right\}$$

T37) $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$

$$A^T A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\chi_{A^TA}(\lambda) = (\lambda-3)(\lambda-1)$$

$$E_3: \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle \quad E_1: \left\langle \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\rangle$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow V = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\sigma_1 = \sqrt{3}, \quad \sigma_2 = 1$$

$$u_1 = \frac{1}{\sigma_1} \cdot A \cdot v_1 = \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{6}} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$u_2 = \frac{1}{\sigma_2} \cdot A \cdot v_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$u_3 = u_1 \times u_2 = \frac{1}{\sqrt{6}} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2\sqrt{3}} \cdot \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow U = \frac{1}{\sqrt{6}} \cdot \begin{pmatrix} 1 & -\sqrt{3} & \sqrt{2} \\ 2 & 0 & -\sqrt{2} \\ 1 & \sqrt{3} & \sqrt{2} \end{pmatrix}, \quad V = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \left(= \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{pmatrix} \right)$$