T33)

a) $A = \begin{pmatrix} 1 & 4 \\ -2 & 7 \end{pmatrix}$

$\chi_A(\lambda) = \det\left(\begin{pmatrix} \lambda-1 & -4 \\ 2 & \lambda-7 \end{pmatrix}\right) = \lambda^2 - 8\lambda + 15 = (x-5)(x-3)$

$\Rightarrow \lambda_1 = 5, \quad \lambda_2 = 3$

$E_5$:

$\text{Kern}\left(\begin{pmatrix} 4 & -4 \\ 2 & -2 \end{pmatrix}\right) = \text{kern}\left(\begin{pmatrix} 4 & -4 \\ 0 & 0 \end{pmatrix}\right) = \langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle$

$E_3$:

$\text{kern}\left(\begin{pmatrix} 2 & -4 \\ 2 & -4 \end{pmatrix}\right) = \text{Kern}\left(\begin{pmatrix} 2 & -4 \\ 0 & 0 \end{pmatrix}\right) = \langle \begin{pmatrix} 2 \\ 1 \end{pmatrix} \rangle$

$\Rightarrow$ Es gilt $m_a(3) = m_g(3) = 1$ und $m_a(5) = m_g(5) = 1$

$\Rightarrow \quad D = \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$

ODER

$D = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix} \quad S = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$\chi_A(\lambda) = (\lambda-1)\cdot\lambda - 1 = \lambda^2 - \lambda - 1$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1-(4\cdot(-1))}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$E_{\lambda_1} : \operatorname{Kern}\left(\begin{pmatrix} \lambda_1 - 1 & -1 \\ -1 & \lambda_1 \end{pmatrix}\right) = \operatorname{Kern}\left(\begin{pmatrix} -1 & \lambda_1 \\ 0 & 0 \end{pmatrix}\right) = \left\langle \begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix} \right\rangle$$

$$E_{\lambda_2} : \operatorname{kern}\left(\begin{pmatrix} \lambda_2 - 1 & -1 \\ -1 & \lambda_2 \end{pmatrix}\right) = \operatorname{Kern}\left(\begin{pmatrix} -1 & \lambda_2 \\ 0 & 0 \end{pmatrix}\right) = \left\langle \begin{pmatrix} \lambda_2 \\ 1 \end{pmatrix} \right\rangle$$

$$\Rightarrow S = \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix} \quad D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

T34) $a_0 = 0$

$a_1 = 1$

$a_{n+1} = a_n + a_{n-1}$

$a_n = a_n$

$$\Rightarrow \begin{pmatrix} a_{n+1} \\ a_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}}_{\text{aus T33b)}} \begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \ldots \cdot \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n}_{\text{zerlege in Diagonalisierung}} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\left(\underbrace{\begin{pmatrix} \lambda_2 & \lambda_2 \\ 1 & 1 \end{pmatrix}}_{S} \cdot \underbrace{\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}_{D} \cdot \underbrace{\frac{1}{\lambda_1 - \lambda_2} \cdot \begin{pmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{pmatrix}}_{S^{-1}}\right)^n \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$(SDS^{-1})^n \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = SD\underbrace{S^{-1}S}_{I_2}DS^{-1} \ldots SDS^{-1} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = SD^nS^{-1} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{pmatrix}\cdot\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}^n \underline{\begin{pmatrix}1 & -\lambda_2\\ -1 & \lambda_1\end{pmatrix}\cdot\begin{pmatrix}1\\ 0\end{pmatrix}} =$$

$$\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{pmatrix}\cdot \underline{\begin{pmatrix}\lambda_1^n & 0\\ 0 & \lambda_2^n\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}} =$$

$$\frac{1}{\lambda_1-\lambda_2}\underline{\begin{pmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{pmatrix}\cdot\begin{pmatrix}\lambda_1^n\\ -\lambda_2^n\end{pmatrix}} = \frac{1}{\lambda_1-\lambda_2}\cdot\begin{pmatrix}\lambda_1^{n+1}-\lambda_2^{n+1}\\ \lambda_1^n-\lambda_2^n\end{pmatrix} = \begin{pmatrix}a_{n+1}\\ a_n\end{pmatrix}$$

$$\Rightarrow a_n = \frac{1}{\lambda_1-\lambda_2}\cdot(\lambda_1^n-\lambda_2^n) = \frac{1}{\sqrt{5}}\cdot\left(\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

T35)

$$W = \begin{pmatrix}1 & 1 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix} \Rightarrow S = \begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$$

$$G := \left(1-\frac{1}{2}\right)\cdot S + \frac{1}{2}\cdot\begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{2}{3} & \frac{1}{6} & \frac{1}{6}\\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6}\end{pmatrix} = \frac{1}{6}\cdot\begin{pmatrix}2 & 2 & 2\\ 4 & 1 & 1\\ 1 & 4 & 1\end{pmatrix}$$

$$1\cdot 1^T = \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$$

$$\rightarrow \cdot \frac{1}{3} = \begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{pmatrix}$$

$\Rightarrow$ Vektor $p$:

$G^T$ den EV zu EW 1

$$\Rightarrow \ker\left(\frac{1}{6}\cdot\begin{pmatrix}2&4&1\\2&1&4\\2&1&1\end{pmatrix}-\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right)=$$

$$\ker\left(\begin{pmatrix}2&4&1\\2&1&4\\2&1&1\end{pmatrix}-\begin{pmatrix}6&0&0\\0&6&0\\0&0&6\end{pmatrix}\right)=\ker\left(\begin{pmatrix}-4&4&1\\2&-5&4\\2&1&-5\end{pmatrix}\right)\begin{matrix}\\ -1\\ +2\end{matrix}=$$

$$\ker\left(\begin{pmatrix}0&6&-9\\0&-6&9\\2&1&-5\end{pmatrix}\right)\begin{matrix}\\ +1\\ \\ \end{matrix}=\ker\left(\begin{pmatrix}0&6&-9\\0&0&0\\2&1&-5\end{pmatrix}\right)\begin{matrix}:3\\ \\ \cdot 2\end{matrix}=\ker\left(\begin{pmatrix}0&2&-3\\0&0&0\\4&2&-10\end{pmatrix}\right)\begin{matrix}\\ -1\\ \\ \end{matrix}=\ker\left(\begin{pmatrix}0&2&-3\\0&0&0\\4&0&-7\end{pmatrix}\right)=\left\langle\begin{pmatrix}7\\6\\4\end{pmatrix}\right\rangle$$

$$\Rightarrow p=\frac{1}{7+6+4}\cdot\begin{pmatrix}7\\6\\4\end{pmatrix}=\frac{1}{17}\cdot\begin{pmatrix}7\\6\\4\end{pmatrix}$$