$$B = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}, \quad B' = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$$

$$C = \left\{ \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\}, \quad C' = \left\{ \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\\1\end{pmatrix} \right\}$$

$$\varphi\left(\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\right) = \begin{pmatrix}x_1-x_2\\-x_2+3x_3\\2x_1\end{pmatrix}, \quad \psi\left(\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\right) = \begin{pmatrix}2x_1+2x_3\\2x_1+x_2-x_3\\2x_1+x_2\\2x_1+3x_3\end{pmatrix}$$

a) $M_B(\varphi)$:

$$\varphi\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\\2\end{pmatrix}, \quad \varphi\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right) = \begin{pmatrix}-1\\-1\\0\end{pmatrix}, \quad \varphi\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = \begin{pmatrix}0\\3\\0\end{pmatrix}$$

$$M_B(\varphi) = \begin{pmatrix}1 & -1 & 0\\0 & -1 & 3\\2 & 0 & 0\end{pmatrix}$$

$M_C^B(\psi)$:

$$\psi\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\right) = \begin{pmatrix}2\\2\\2\\2\end{pmatrix}, \quad \psi\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right) = \begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \quad \psi\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = \begin{pmatrix}2\\-1\\0\\3\end{pmatrix}$$

$$M_C^B(\psi) = \begin{pmatrix}2 & 0 & 2\\2 & 1 & -1\\2 & 1 & 0\\2 & 0 & 3\end{pmatrix}$$

$$M_C^B(\psi \circ \varphi) = M_C^B(\psi) \cdot M_B(\varphi) = \begin{pmatrix}2 & 0 & 2\\2 & 1 & -1\\2 & 1 & 0\\2 & 0 & 3\end{pmatrix} \cdot \begin{pmatrix}1 & -1 & 0\\0 & -1 & 3\\2 & 0 & 0\end{pmatrix} = \begin{pmatrix}6 & -2 & 0\\0 & -3 & 3\\2 & -3 & 3\\8 & -2 & 0\end{pmatrix}$$

b) $T_B^{B'} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$T_{B'}^{B} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

$T_C^{C'} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$T_{C'}^{C} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$\uparrow$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \underline{0} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \underline{0} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \underline{-1} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \underline{1} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

c) $M_{B'}(\varphi) = T^{B}_{B'} \cdot M_B(\varphi) \cdot T^{B'}_{B} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 3 \\ 2 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} =$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -2 \\ -2 & -3 & 0 \\ 2 & 2 & 2 \end{pmatrix}$$

$M^{B'}_{C'}(\varphi) = T^{C}_{C'} \cdot M^{B}_{C}(\varphi) \cdot T^{B'}_{B} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 2 \\ 2 & 1 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} =$

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & 4 \\ 2 & 3 & 2 \\ 2 & 3 & 3 \\ 2 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 0 & 0 & -1 \\ 0 & 1 & -2 \\ 2 & 1 & 5 \end{pmatrix}$$

2b) $E = \{1, x, x^2\}$, $B = \{2x+1, 7x+3, x^2+x+2\}$

a) T_E^B :

$$2x+1 = 1 \cdot (1) + 2 \cdot (x) + 0 \cdot (x^2)$$
$$7x+3 = 3 \cdot (1) + 7 \cdot (x) + 0 \cdot (x^2)$$
$$x^2+x+2 = 2 \cdot (1) + 1 \cdot (x) + 1 \cdot (x^2)$$

$$T_E^B = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 7 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad T_B^E = \left(T_E^B\right)^{-1}$$

b

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 2 & 7 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \cdot (2) \rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \cdot 3 \;\; \cdot (2) \rightsquigarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & -2 & 1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \cdot (-3) \rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -11 \\ 0 & 1 & 0 & -2 & 1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\underbrace{\begin{pmatrix} 7 & -3 & -11 \\ -2 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}}_{\left(T_E^B\right)^{-1} = T_B^E}$$

$$\Rightarrow T_B^E = \begin{pmatrix} 7 & -3 & -11 \\ -2 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

b) $f = a_0 + a_1 x + a_2 x^2$ zur Basis B

f zur Basis E : $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$

$$\begin{pmatrix} 7 & -3 & -11 \\ -2 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 7a_0 - 3a_1 - 11a_2 \\ -2a_0 + a_1 + 3a_2 \\ a_2 \end{pmatrix}$$

$$(7a_0 - 3a_1 - 11a_2)\cdot(2x+1) + (-2a_0 + a_1 + 3a_2)\cdot(7x+3) + a_2\cdot(x^2+x+2) =$$

$$\cancel{14a_0}x \underline{+7a_0} - \cancel{14}a_0 x \underline{-6a_0} \underline{-6a_1x} - \cancel{3a_1} + \underline{7a_1x} + \cancel{3a_1} - \cancel{22a_2}x - \cancel{11a_2} + \cancel{21a_2}x + \cancel{9a_2} \underline{+a_2x^2} + \cancel{a_2}x + \cancel{2a_2}$$

$$a_0 + a_1x + a_2x^2$$