30a)

$$\chi_A(\lambda) = \det(\lambda I_2 - A) =$$

$$\det\begin{pmatrix}\lambda-1 & -4\\ 2 & \lambda-7\end{pmatrix} = (\lambda-1)(\lambda-7)+8 = \lambda^2-8\lambda+15 = (\lambda-3)(\lambda-5)$$

$$\Rightarrow \lambda_1 = 3 \qquad \lambda_2 = 5$$

$$E_{\lambda_1} = \mathrm{Kern}(\lambda_1 I_2 - A): \left(\begin{array}{cc|c} 3-1 & -4 & 0\\ 2 & 3-7 & 0\end{array}\right) = \left(\begin{array}{cc|c} 2 & -4 & 0\\ 2 & -4 & 0\end{array}\right) \xrightarrow{-1} \left(\begin{array}{cc|c} 2 & -4 & 0\\ 0 & 0 & 0\end{array}\right) \quad \mathbb{L} = \left\langle \begin{pmatrix}2\\1\end{pmatrix}\right\rangle \Rightarrow E_3 = \left\langle \begin{pmatrix}2\\1\end{pmatrix}\right\rangle$$

$$E_{\lambda_2} = \mathrm{Kern}(\lambda_2 I_2 - A): \left(\begin{array}{cc|c} 5-1 & -4 & 0\\ 2 & 5-7 & 0\end{array}\right) = \left(\begin{array}{cc|c} 4 & -4 & 0\\ 2 & -2 & 0\end{array}\right) \xrightarrow{\cdot -\frac{1}{2}} \left(\begin{array}{cc|c} 4 & -4 & 0\\ 0 & 0 & 0\end{array}\right) \Rightarrow \mathbb{L} = \left\langle \begin{pmatrix}1\\1\end{pmatrix}\right\rangle \Rightarrow E_5 = \left\langle \begin{pmatrix}1\\1\end{pmatrix}\right\rangle$$

$$\Rightarrow S = \begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}, \quad D = \begin{pmatrix}3 & 0\\ 0 & 5\end{pmatrix} \quad \text{mit } A = SDS^{-1}$$

b) $$\chi_A(\lambda) = \det(\lambda \cdot I_2 - A) = \det\begin{pmatrix}\lambda-1 & -1\\ -1 & \lambda\end{pmatrix} = (\lambda-1)\lambda - 1 = \lambda^2 - \lambda - 1$$

$$\Rightarrow \lambda_1 = \frac{1+\sqrt{5}}{2} \qquad \lambda_2 = \frac{1-\sqrt{5}}{2}$$

$$E_{\lambda_1} = \mathrm{Kern}(\lambda_1 I_2 - A): \left(\begin{array}{cc|c} \lambda_1-1 & -1 & 0\\ -1 & \lambda_1 & 0\end{array}\right) \xrightarrow{\lambda_1} \left(\begin{array}{cc|c} \lambda_1-1 & -1 & 0\\ 0 & 0 & 0\end{array}\right) \quad \mathbb{L} = \left\langle \begin{pmatrix}\lambda_1\\1\end{pmatrix}\right\rangle \Rightarrow E_{\lambda_1} = \left\langle \begin{pmatrix}\lambda_1\\1\end{pmatrix}\right\rangle$$

$$\text{NR: } \left(\frac{1+\sqrt{5}}{2} - 1\right) \cdot \frac{1+\sqrt{5}}{2} = \frac{1+2\sqrt{5}+5}{4} - \frac{1+\sqrt{5}}{2} = \frac{2}{2} = 1$$

$$E_{\lambda_2} = \text{Kern}(\lambda_2 I_2 - A): \left(\begin{array}{cc|c} \lambda_2 - 1 & -1 & 0 \\ -1 & \lambda_2 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} -1 & \lambda_2 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow L = \left\langle \begin{pmatrix} \lambda_2 \\ 1 \end{pmatrix} \right\rangle \Rightarrow E_{\lambda_2} = \left\langle \begin{pmatrix} \lambda_2 \\ 1 \end{pmatrix} \right\rangle$$

$$S = \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \qquad A = SDS^{-1}$$

$$a_{n+1} = a_n + a_{n-1} \qquad a_n = a_n$$

T31)
$$\begin{pmatrix} a_{n+1} \\ a_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix} = \ldots = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \cdot \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} =$$

$$\underline{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow a_n = \begin{pmatrix} 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \end{pmatrix} \cdot S D^n S^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix} \cdot \frac{1}{\lambda_1 - \lambda_2} \cdot \begin{pmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\frac{1}{\lambda_1 - \lambda_2} \cdot \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \underline{\begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}} = \frac{1}{\lambda_1 - \lambda_2} \cdot \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \lambda_1^n \\ -\lambda_2^n \end{pmatrix} =$$

$$\frac{1}{\lambda_1-\lambda_2}\cdot\left(\lambda_1^n-\lambda_2^n\right) = \frac{1}{\sqrt{5}}\cdot\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right) = a_n$$

T32) a) $\chi_A = \chi_{A^T}$

$$\chi_A = \det(\lambda\cdot I_n - A) \overset{\substack{\text{Transponieren}\\ \text{ändert}\\ \text{det nicht}}}{=} \det\left((\lambda\cdot I_n - A)^T\right) = \det\left(\lambda\cdot I_n^T - A^T\right) = \det\left(\lambda\cdot I_n - A^T\right) = \chi_{A^T} \checkmark$$

$\Rightarrow$ A und A^T haben gleiche EW

b) Weil $\chi_A = \chi_{A^T} \Rightarrow m_a(\lambda, A) = m_a(\lambda, A^T)$

$$m_g(\lambda, A) = \dim(\mathrm{Kern}(\lambda\cdot I_n - A)) = n - rg(\lambda\cdot I_n - A) \overset{\substack{\text{Spaltenrang}\\ =\\ \text{Zeilenrang}}}{=} n - rg\left((\lambda\cdot I_n - A)^T\right) =$$

$$n - rg\left(\lambda I_n^T - A^T\right) = n - rg\left(\lambda I_n - A^T\right) = \dim\left(\mathrm{Kern}(\lambda\cdot I_n - A^T)\right) = m_g(\lambda, A^T) \checkmark$$

$\Rightarrow m_a(\lambda, A) = m_a(\lambda, A^T)$ $\forall\lambda$ EW von A und A^T

$\Rightarrow m_g(\lambda, A) = m_g(\lambda, A^T)$ ——— " ———

c) Nein: Beispiel $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\lambda_{1,2} = 1$

$$E_1(A) = \mathrm{Kern}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\rangle$$

$$E_1(A^T) = \mathrm{Kern}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle$$