$$\varphi\left(\begin{pmatrix}1\\4\end{pmatrix}\right)=\begin{pmatrix}1\\0\end{pmatrix}$$
$$\varphi\left(\begin{pmatrix}2\\7\end{pmatrix}\right)=\begin{pmatrix}3\\1\end{pmatrix}$$

$$\varphi\left(\begin{pmatrix}x\\y\end{pmatrix}\right)$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\lambda\cdot\underbrace{\begin{pmatrix}1\\4\end{pmatrix}}_{b_1}+\mu\cdot\underbrace{\begin{pmatrix}2\\7\end{pmatrix}}_{b_2}$$

$$\left(\begin{array}{cc|c}1&2&x\\4&7&y\end{array}\right)\cdot(-4)\rightsquigarrow\left(\begin{array}{cc|c}1&2&x\\0&-1&y-4x\end{array}\right)\begin{matrix}2\\\cdot(-1)\end{matrix}\rightsquigarrow\left(\begin{array}{cc|c}1&0&-7x+2y\\0&1&4x-y\end{array}\right)$$

$$\lambda=2y-7x\quad\mu=4x-y$$

$$\varphi\left(\begin{pmatrix}x\\y\end{pmatrix}\right)=\varphi\left((2y-7x)\cdot b_1+(4x-y)\cdot b_2\right)=$$
$$(2y-7x)\cdot\varphi(b_1)+(4x-y)\cdot\varphi(b_2)=$$
$$(2y-7x)\cdot\begin{pmatrix}1\\0\end{pmatrix}+(4x-y)\cdot\begin{pmatrix}3\\1\end{pmatrix}=\begin{pmatrix}5x-y\\4x-y\end{pmatrix}$$

$$\begin{pmatrix}5\cdot3-3\\4\cdot3-3\end{pmatrix}=\begin{pmatrix}12\\9\end{pmatrix}\neq\begin{pmatrix}1\\1\end{pmatrix}$$

$\Rightarrow\psi$ keine lin. Abb.

$f,g\in V,\quad\lambda\in\mathbb{R}$

$$\varphi(\lambda f+g)=(\lambda f+g)(2)\cdot x+(\lambda f+g)'=$$
$$\lambda\cdot f(2)\,x+g(2)\cdot x+\lambda f'+g'=$$
$$\lambda\cdot f(2)\cdot x+\lambda f'+g(2)\cdot x+g'=$$

$\lambda \cdot (f(1) \cdot x + f') + \varphi(g) = \lambda \cdot \varphi(f) + \varphi(g)$ ✓

$E = \{1, x, x^2\}$ $\quad F = \{1, x+1, x^2+1\}$

$-1 \cdot 1 + 1 \cdot (x+1) + 0\,(x^2+1)$

$\varphi(1) = x = 0\,1 + 1 \cdot x + 0 \cdot x^2 \quad (0,1,0) \quad (-1,1,0)$

$\varphi(x) = 2x+1 = 1 \cdot 1 + 2 \cdot x + 0\,x^2 \quad (1,2,0)$

$\varphi(x^2) = 4x + 2x = 6x = 0 \cdot 1 + 6\,x + 0 \cdot x^2 \quad (0,6,0)$

$D_E(\varphi) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix} \qquad D_{F,E} \qquad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle$

$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix} \qquad U = \left\langle \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}, \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} \right\rangle$

Kern(φ):

$\left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 1 & 2 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \cdot(-2) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad U = \left\langle \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

$\text{Kern}(\varphi) = \langle \{-6 \cdot 1 + x^2\} \rangle = \langle \{x^2 - 6\} \rangle$

$\dim(\text{Bild}(\varphi)) = \dim(V) - \dim(\text{Kern}(\varphi)) = 3 - 1 = 2$

$$\left\{\begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\2\\0\end{pmatrix}\right\} \Rightarrow \langle\{x,\ 1+2x\}\rangle = \mathrm{Bild}(\varphi)$$

$ad-bc \neq 0 \iff A$ ist invertierbar und $A^{-1} = \frac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

"$\Rightarrow$" Es sei $ad-bc \neq 0$

$$B = \frac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A \cdot B = \frac{1}{ad-bc} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{ad-bc} \cdot \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

"$\Leftarrow$" $a \neq 0$ (A ist invertierbar wenn $\mathrm{rang}(A) = 2$)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow{-\frac{c}{a}} \begin{pmatrix} a & b \\ 0 & d - \frac{bc}{a} \end{pmatrix}$$

$\mathrm{rang}(A) = 2 \Rightarrow d - \frac{bc}{a} \neq 0$

weil $a \neq 0$: $a \cdot \left(d - \frac{bc}{a}\right) = ad - bc \neq 0$

1. Fall: $a \neq 0$

$\begin{pmatrix} 0 & b \\ c & d \end{pmatrix}$ $\quad \text{rang}(A) = 2 \Rightarrow b \neq 0, \; c \neq 0$

$\Rightarrow \underbrace{ad}_{=0} - bc = -bc \neq 0 \quad \checkmark$

$\det(A) = ad - bc \quad \left(\text{für } A \in K^{2\times 2} \right)$

$\det(A) \neq 0$ wenn A invertierbar ist

$\det(A)^{-1} \cdot \begin{pmatrix} a & -d \\ -b & c \end{pmatrix}$

$\det(B) = 3 \cdot 4 - 1 \cdot 2 = 10 \mod 5 = 0$ $\quad a \cdot a^{-1} = 1 \quad a, a^{-1} \in \mathbb{F}_5$

$$\det(C) = 4 - 2 = 2$$

$$C^{-1} = 2^{-1} \cdot \begin{pmatrix} 4 & -1 \\ -2 & 1 \end{pmatrix} = 3 \cdot \begin{pmatrix} 4 & 4 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 4 & 3 \end{pmatrix}$$

$$2^{-1} = \frac{1}{2} \cdot 2 = 1$$

$$2^{-1}: \quad 2 \cdot 2^{-1} = 1 = 2 \cdot 3 = \text{"}6\text{"} \bmod 5 = 1$$