7.1 a)

a_1 : ~~b_3~~ > b_2 > b_1 > b_4

a_2 : b_4 > b_1 > b_3 > b_2

a_3 : ~~b_4~~ > ~~b_2~~ > ~~b_3~~ > b_1

a_4 : ~~b_2~~ > b_3 > b_4 > b_1

1: $(a_1, b_3), (a_1, b_4), (a_3, b_2), (a_4, b_2)$

2: (a_3, b_3)

3: $(a_1, b_2), (a_4, b_3)$

4: (a_3, b_1)

$\Rightarrow a_1 \leftrightarrow b_2,\ a_2 \leftrightarrow b_4,\ a_3 \leftrightarrow b_1,\ a_4 \leftrightarrow b_3$

b)

1: $(a_1, b_4), (a_2, b_5), (a_3, b_5), (a_4, b_4), (a_5, b_3)$

2: $(a_1, b_1), (a_2, b_3), (a_5, b_5)$

3: (a_3, b_2)

$\Rightarrow a_1 \leftrightarrow b_1,\ a_2 \leftrightarrow b_3,\ a_3 \leftrightarrow b_2,\ a_4 \leftrightarrow b_4,\ a_5 \leftrightarrow b_5$

7.2 a)

$$A^2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 3 & 3 & 1 \\ 2 & 2 & 3 & 1 \\ 1 & 3 & 2 & 2 \\ 1 & 3 & 3 & 1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 3 & 5 & 5 & 3 \\ 2 & 6 & 5 & 3 \\ 3 & 5 & 6 & 2 \\ 3 & 5 & 5 & 3 \end{pmatrix}$$

b) $T = 4$

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} = \frac{1}{2} \cdot A$$

$$Q = \frac{1}{5} \cdot \frac{1}{16} \cdot \begin{pmatrix} 25 & 23 & 23 & 9 \\ 14 & 34 & 23 & 9 \\ 9 & 23 & 34 & 14 \\ 9 & 23 & 23 & 25 \end{pmatrix}$$

$$Q = \frac{1}{5} \cdot \left(P^0 + \frac{1}{2}A + \frac{1}{4}A^2 + \frac{1}{8}A^3 + \frac{1}{16}A^4 \right)$$

$$P^2 = \frac{1}{2} \cdot A \cdot \frac{1}{2} \cdot A = \frac{1}{4}A^2$$

$$P^3 = \frac{1}{2} \cdot A \cdot \frac{1}{4}A^2 = \frac{1}{8}A^3$$

$$P^4 = \frac{1}{4} \cdot A \cdot \frac{1}{4}A^2 = \frac{1}{16}A^4$$

$$P^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$