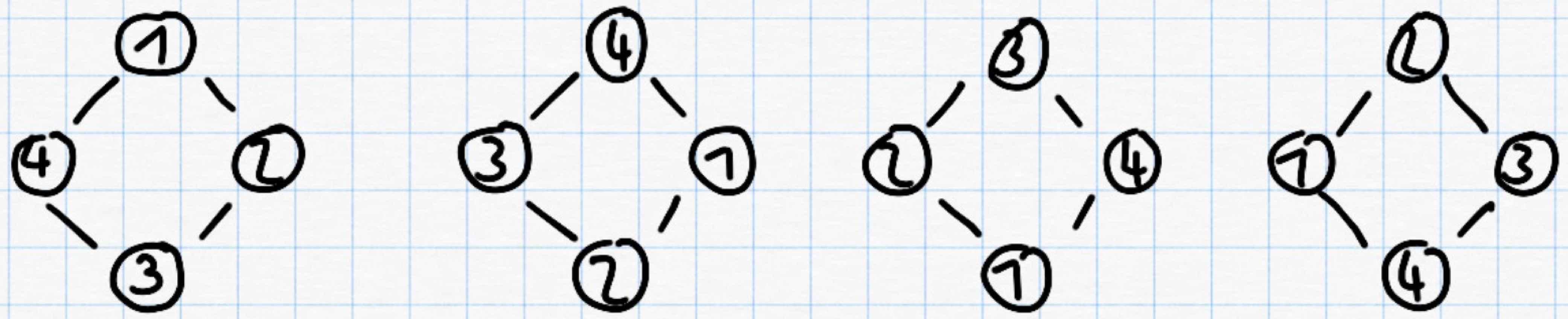
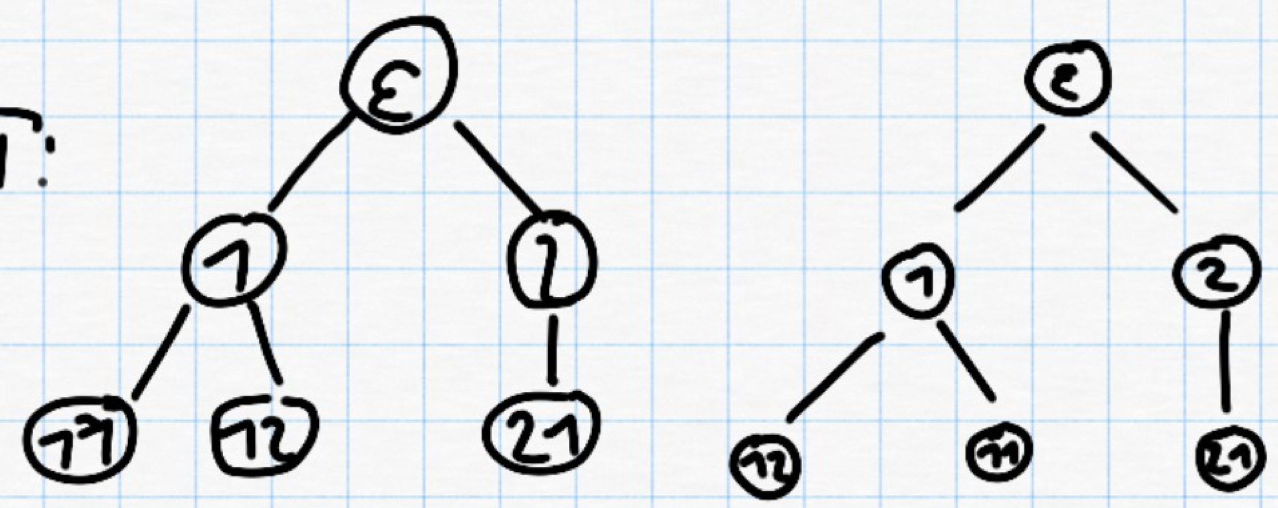
5.2

C_4:



$$\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix} \quad \begin{pmatrix}1&2&3&4\\4&1&2&3\end{pmatrix} \quad \begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix} \quad \begin{pmatrix}1&2&3&4\\2&3&4&1\end{pmatrix}$$

T:



$$\begin{pmatrix}\varepsilon&1&2&21&12&11\\\varepsilon&1&2&21&11&12\end{pmatrix}$$

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

Basis: $n=0$. $\sum_{k=0}^{0} k = 0$ ✓

$\frac{0(0+1)}{2} = 0$ ✓

Sei n beliebig, aber fest.

Ann.: $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$

Beh: $\sum_{k=0}^{n+1} k = \frac{(n+1)((n+1)+1)}{2}$

Beweis: $\sum_{k=0}^{n+1} k = \frac{(n+1)(n+2)}{2}$

$\left(\sum_{k=0}^{n} k\right) + (n+1) = "$

Einsetzen der Annahme:

$\frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2}$

$\frac{n(n+1)}{2} + \frac{2(n+1)}{2} = "$

$\frac{n^2+n+2n+2}{2} = "$

$\frac{(n+1)(n+2)}{2} = "$ $\square$

$\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$ für alle $x \in \mathbb{R} \setminus \{1\}$

Basis: $n=0$: $\sum_{k=0}^{0} x^k \overset{!}{=} x^0 = 1$

Sei n bel., aber fest

Ann.: $\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$

Beh.: $\sum_{k=0}^{n+1} x^k = \frac{1-x^{n+2}}{1-x}$

$x \in \mathbb{R}\setminus\{1\}$

$$\sum_{k=0}^{n} x^k + x^{n+1} = \frac{1-x^{n+2}}{1-x}$$

Ann. einsetzen:

$$\frac{1-x^{n+1}}{1-x} + x^{n+1} \stackrel{"}{=}$$

$$\frac{1-x^{n+1}+(1-x)x^{n+1}}{1-x} \stackrel{"}{=}$$

$$\frac{1-x^{n+1}+x^{n+1}-x^{n+2}}{1-x} \stackrel{"}{=}$$

$$\frac{1-x^{n+2}}{1-x} \stackrel{"}{=}$$

$\square$

c) $$F_n = \frac{\phi^n - (-\phi)^{-n}}{\phi - (-\phi)^{-1}}$$

Basis: $n=0$

$F_0 := 0$

$$\frac{\phi^0 - (-\phi)^0}{\phi + \phi^{-1}} = \frac{1-1}{\dots} = 0 \quad \checkmark$$

$n=1$

$F_1 := 1$ ✓

$$\frac{\phi - (-\phi)^{-1}}{\phi - (-\phi)^{-1}} = 1 \quad ✓$$

Behauptung: $$\frac{\phi^{n+1} - (-\phi)^{-(n+1)}}{\phi - (-\phi)^{-1}} = F_n + F_{n-1}$$

Beweis: $$\frac{\phi^{n-1}(\phi^2) - ((-\phi)^{-(n-1)}(\phi^2))}{\phi - (-\phi)^{-1}} =$$

$$1 + 2^{-1} = 1 + \frac{1}{2}$$

$$1 - (-2)^{-1} = 1 - (-\frac{1}{2})$$

$$\frac{\phi^{n-1}(1+\phi) - (-\phi)^{-(n-1)}(1+\phi)}{\ldots}$$

$$\frac{\phi^{n-1} + \phi^{n} - \left(-\phi^{-(n-1)} - \phi^{n}\right)}{}$$

$$\frac{\phi^{n-1} - \left(-\phi^{-(n-1)}\right)}{\ldots} + \frac{\phi^{n} - \left(-\phi^{n}\right)}{\ldots}$$

Induktionsannehme einsetzen:

$$F_{n-1} + F_{n} = F_{n+1}$$